Numerical Study of Laminar Free Convection Heat Transfer inside Porous Media -Filled Triangular Enclosure

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Abstract:

The natural convection heat transfer in a porous media filled and isothermally heated from the bottom wall of triangular enclosure is analyzed using finite element software package (FLEXPDE). Darcy's law was used to write equations of porous media . The curved bottom wall shape, with Radii R= 0.8 , 1 and 1.5, was applied to a triangular enclosure. The boundary condition of the vertical wall is isothermal and of the inclined wall is adiabatic. The study was performed for different Rayleigh numbers ($100 \le Ra \le 1000$) and aspect ratios ($0.4 \le AR \le 1$) . Numerical results are presented in terms of streamlines, isotherms and Nusselt numbers. It was observed that heat transfer enhancement was formed with increasing Rayleigh number and aspect ratio . A comparison of the flow field and isotherm field is made with that obtained by [11], which revealed a good agreement .

دراسة نظرية لعملية انتقال الحرارة بالحمل الحر الطباقي المستقر داخل غلاف مثلث مملوء بالمادة المتسامية

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الخلاصة:

تمت في هذا البحث دراسة عددية لعملية إنتقال الحرارة بالحمل الحر بين السطح السفلي المسخن عند درجة حرارة ثابتة لغلاف مثلث الشكل وبين المادة المتسامية التي تملأ الغلاف . استخدمت الحقيبة البرمجية التي تعمل بطريقة العناصر المحددة (FLEXPDE) لحل منظومة المعادلات الحاكمة لعملية انتقال الحرارة في الوسط المتسامي والتي تتبع لقانون دارسي . الجدار السفلي للغلاف يكون بشكل منحني عند قيم مختلفه لنصف القطر R=0.8 و R=1 و R=1.5 . الشرط الحدي للجدار المائل يكون معزول اما الجدار العمودي فيكون مثبت عند درجة حرارة المحيط . الدراسة انجزت لرقم رايلي Ra يتراوح من 100 إلى 1000 ولنسبة ثابتة من ارتفاع المثلث الى القاعدة AR تتراوح من 0.4 إلى 1 .أظهرت النتائج التي تمثلت بواسطة خطوط الجريان و خطوط التحارر ورقم نسلت بأن رقم نسلت يزداد بازدياد كل من Ra , AR وكذلك نصف القطر R . قورنت النتائج المستحصلة مع ما منشور في المصدر [11] وأظهرت تقاربا" جيدا" .

1.Introduction

An analysis of a fluid saturated porous media is important in engineering due to its wide application areas in geophysics , heat exchangers , groaned coupled heat pumps, solar collectors, reactors, grain storage, oil extraction , fluid flow in geothermal reservoirs, iron blast furnaces , solidification of castings etc. Natural convection phenomena in different shaped enclosures filled with fluid saturated porous media is important in applications due to the knowledge of flow field and temperature distribution which help to design high efficiency thermal systems. In the past years, most of the researches focused on the investigation of natural convection in porous square or rectangular enclosures with constant temperature or heat flux boundary conditions as reported in the literature by Bejan [1] ,Gross et al.[2] , Manole and Lage [3] , Goyeau et al.[4] , Baytas and Pop[5,6]. Basak et al. [7] studied numerically the free convection flows in a square cavity filled with a porous matrix for various boundary conditions with wide range of parameters ($10^3 \leq Ra \leq 10^6$) and ($0.71 \leq Pr \leq 10$) . Results of non-uniform heating for the bottom wall produce greater heat transfer rate at the center of the bottom wall than the uniform heating case for all Rayleigh numbers . The shape of enclosure can be different than a rectangular geometry such trapezoidal [8,9] . A few studies on natural convection in triangular enclosures filled with porous media . Basak et al. [10] , studied numerically the natural convection in an isosceles triangular enclosure filled with a porous matrix for varying boundary conditions and a wide range of parameters , Darcy number ($10^{-5} \leq Da \leq 10^{-3}$) , ($10^3 \leq Ra \leq 10^6$) and ($0.026 \leq Pr \leq 10$) . Numerical results are presented in terms of stream functions, temperature profiles and Nusselt numbers. Yasin et al. [11] , studied the natural convection heat transfer in a porous medium filled and non-isothermally heated from the bottom wall of a triangular enclosure , and analyzed the results using finite difference techniques . The study was performed for ($100 \leq Ra \leq 1000$) . The aim of the present paper is to provide a solution procedure for the natural convection heat transfer in porous media filled right- angle triangular enclosure using finite element software package

(Flexpde). A detailed study of the temperature and flow field with detailed analysis on heat transfer evaluation was carried out .The geometry of a right-angle triangular enclosure filled with a fluid saturated porous medium with length of bottom wall L, and height of vertical wall H , as illustrated in Fig.1 .

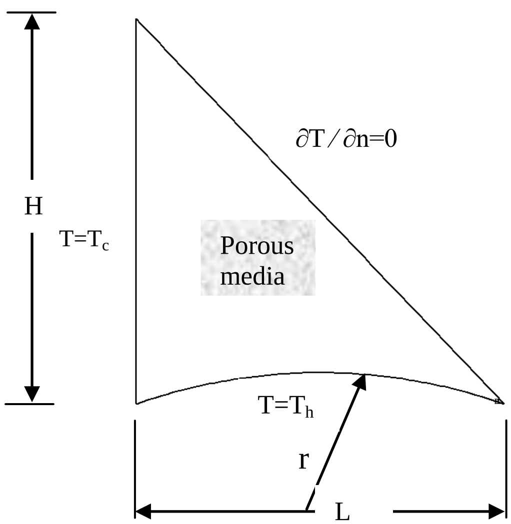

Fig.1 Schematic diagram of the physical domain.

2. Governing equations

The dimensional set of the governing equations of the present problem are the continuity , Darcy law and energy given by eqs(1-3) . Properties of the fluid and the porous medium are constant ; the Boussinesq approximation is valid ; and the viscous drag and inertia terms in the momentum equation are neglected .

The governing equations can be written as follows :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{g\beta k}{\upsilon}\frac{\partial T}{\partial x} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

Where u, v are the velocity components along x- and y- axes , respectively , T is the fluid temperature , k is the permeability of the porous medium and α is the thermal diffusivity of the porous media . The above equations can be written in terms of the stream function defined as :

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x} \qquad (4)$$

Non-dimensional parameters can be given as follows:

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{uL}{\alpha},\ V=\frac{vL}{\alpha},\ \theta=\frac{T-T_C}{T_H-T_C} \qquad (5)$$

The Ra is the Darcy modified Rayleigh number defined as :

$$Ra=\frac{g\beta k(T_H-T_C)}{\upsilon\alpha} \qquad (6)$$

Thus eqs.(1-3) can be written in non-dimensional form a as follows :

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (7)$$

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=-Ra\frac{\partial\theta}{\partial X} \qquad (8)$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2} \qquad (9)$$

The appropriate boundary conditions are as follows:

1- Isothermal surfaces i.e. θ=1 on the bottom walls and θ=0 on the vertical wall.

2-Adiabatic inclined wall $\frac{\partial\theta}{\partial n}=0$

3-No-slip velocity boundary condition, U=V=0 on all solid walls.

The physical quantities of interest in this problem are the local and mean Nusselt numbers which for the bottom wall are given by :

$$Nu_x=-\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \qquad (10)$$

$$Nu_m=\int_0^1 Nu_x dx \qquad (11)$$

3. Numerical solution

In the present study, a finite element software package (Flexpde) [12] is relied on in solving of the nonlinear system of the two equations (8) and (9) . Hence, the continuity equation (7) is used to check the error of the solution through out the grids of the domain.

3.1 Software Validation

To check the validation of a software, the distribution of the values of (∂U/∂X + ∂V/∂Y) over the domain for Ra=100 , AR=1 and R=1.5 is presented in Fig.2. It is clear that the continuity equation is exactly validated, where it as mentioned in section 3, does not contribute in solving the governing equations.

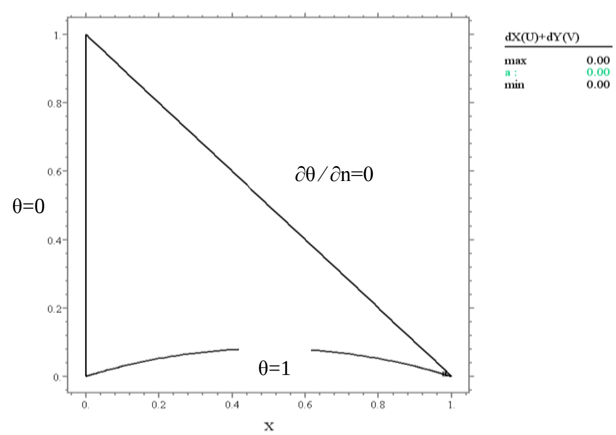

Fig. 2. Validation of continuity equation for Ra=100 , AR=1 and R=1.5

3.2 Code Validation

A computational mode is validated by comparing the results of natural convection in right-angle triangular enclosure with sinusoidal temperature distribution (θ=0.5(1-cos(2πx))) of the flat bottom wall performed by [11] . The boundary conditions of the vertical wall and inclined wall are isothermal , θ=0 .

In the present work numerical predictions have been obtained for flat bottom wall and for the same boundary condition of Yasin et al.[11]. Fig.3 shows the comparison of the flow and thermal fields between the present investigation and Yasin et al. [11] . The results show a very good agreement . From this comparison it can be decided that the current code can be used to predict the flow field for the present problem.

Yasin et al. [11]

Present

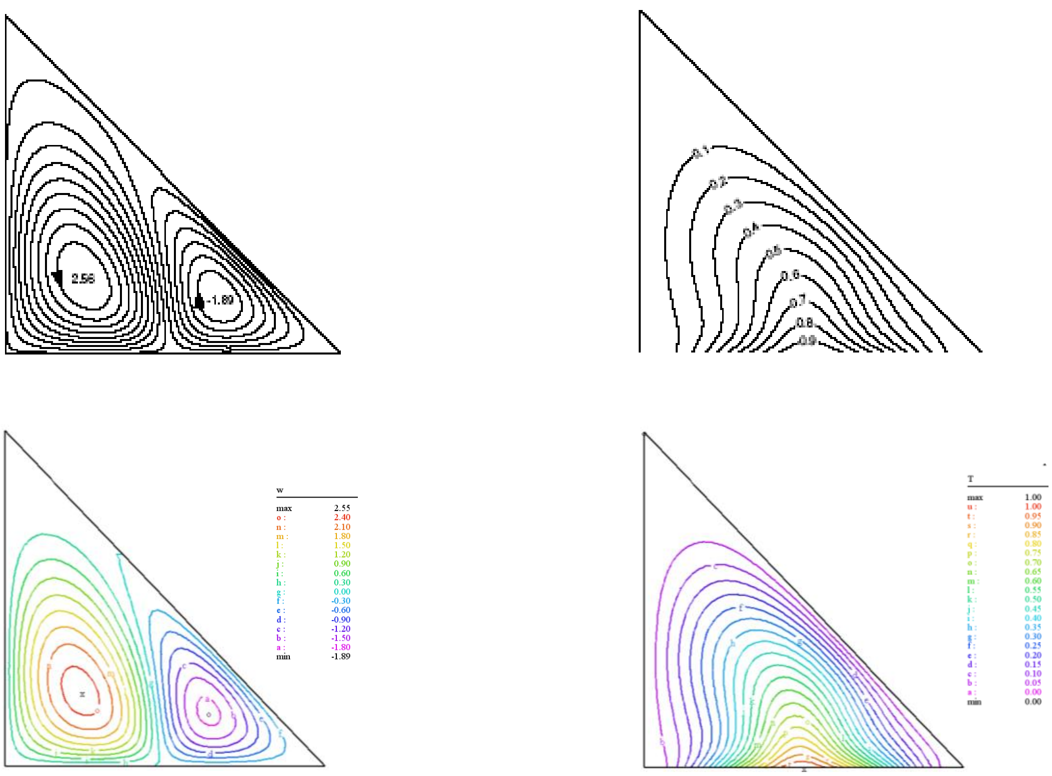

Fig. 3. Comparison of streamlines and isotherms for validation at Ra =100 and AR =1 with the results of Yasin et al. [11]

4. Results and discussion

The results of the flow fields , temperature distribution and heat transfer for porous isothermally heated triangular enclosure are examined in this section. Fig.4 shows the streamlines (on the left) and isotherms (on the right) in a porous triangular enclosure with AR=1 and R=1.5 for different values of

Rayliegh number, the bottom wall is taken as curved wall. For low Rayliegh number, the circulation inside the enclosure is so weak that the viscous forces are dominant over the buoyancy force.The intensity of the recirculation in side the enclosure increases as Rayliegh number increases due to dominant of the buoyancy force which causes the density change near the heated surface that create a growth of the thermal boundary layer. It can be seen that, the single circulation elliptical cell was formed starting in the middle of the enclosure for all Rayleigh numbers considered. The figure also shows the maximum and minimum streamlines values, as given, for each contour the maximum values of streamlines occur near the center of enclosure and they decrease toward the walls. This is because the increasing the effect of the secondary flow near the center of the enclosure . As illustrated in the figure, the isotherms show that for low Rayleigh numbers, the temperature is confined to a part of the bottom wall in the form of a thermal boundary layer, this layer becomes thinner and covers most of the bottom wall when the Rayleigh number is increased. For small Rayleigh numbers, the temperature distribution is almost the same as in the pure conduction case.However , for high Rayleigh number, the growth of the thermal boundary layer is increased and the natural convection effect become dominant instead of the conduction . Because of the presence of the inclined wall and the increasing of the curvature of the bottom wall , the isotherms show wavy variation and no vortices contours are observed. Also it can be seen that , as Rayleigh number increases,the isotherms are observed to have a plum like distribution and have a skewness towered the right corner of the enclosure due to the effect of the free convection . Fig.5 shows the streamlines (on the left) and isotherms(on the right) for AR=1 and R=0.8 , the bottom wall becomes of a high curvature. As given in this case , the trend of the streamlines and isotherms curves is the same as that in Fig.4.The sizes of the elliptical cell shown in Fig.5 are considerably smaller than those of the corresponding case given in Fig.4 which indicate an increase of the flow strength. Also the skewness of streamlines and isotherms toward the right corner is less then that in Fig.4. This is because of the dominant of conduction near the right corner.

The effect of aspect ratio AR, on streamlines and isotherms, was tested for Ra=1000, and the results are illustrated in Fig. 6. The value of aspect ratio with in the range of 0.4-1. For all cases of aspect ratio, one elliptical cell of the streamlines was formed. At high aspect ratio a plum like distribution is shown from the isotherms and its cover the most area of the enclosure due to the increasing the growth of the thermal boundary layer and dominant of the natural convection. The shift of the plum to the corner point decrease with the increases of the aspect ratio this is because the high effect of the conduction at the right part of the bottom heated wall. At high aspect ratio the elliptical cell of vortices is located near the middle of the enclosure, as the aspect ratio gets smaller the location of the cell is shifted toward the vertical wall and the length of the cell becomes smaller due to increasing the effect of the secondary flow .

Variations of local Nusselt number along the hot bottom wall for AR=1 , R=1.5 and different Rayliegh numbers are presented in Fig.7 . As can be seen from this figure, the value of local Nusselt number decreases along the bottom wall towered the right corner. A weak heat transfer from the bottom wall near the right corner region of the enclosure that forms, through most of the heat in the enclosure is transferred from the bottom to the vertical wall. This is due to the dominant conduction heat transfer mechanism prevalent in the right corner of the triangular enclosure. The intersection between the hot bottom wall and the cold vertical wall enhances the natural convection heat transfer. The figure shows, the increase of heat transfer rate from the bottom wall with the increases of Rayliegh number due to increasing the buoyancy effect and dominant the convection heat transfer .

Fig.8, shows the effects of aspect ratio on the variation of local Nusselt number for R=1.5 and Ra=1000. The curves show the same trend as that in Fig.7. As seen from this figure the values of Nusselt numbers are increased with increasing of the aspect ratio. This is because the isotherms are cover most of the cavity space leading to enhancing the convection heat transfer process. The change of the mean Nusselt number with Rayliegh number for various values of bottom wall curvature is presented in Fig.9. For a given Rayliegh number, it can seen that, while the value of R decreases (the curvature of the bottom wall is increased) the amount of heat transfer increases due to the increasing the intensity of the recalculation. It can be seen that the curvature of the bottom wall has a great influence on the mean Nusselt number Nu_m, that represents the heat transfer rate at the base of the enclosure. The greater the curvature, more growth the thermal boundary layer along the heated surface leading to enhancing the heat transfer process from the bottom wall. Fig.(10) shows the increase of the mean Nusselt number with the increases of the aspect ratio, due to the increasing the buoyancy effect then dominant the convection heat transfer.

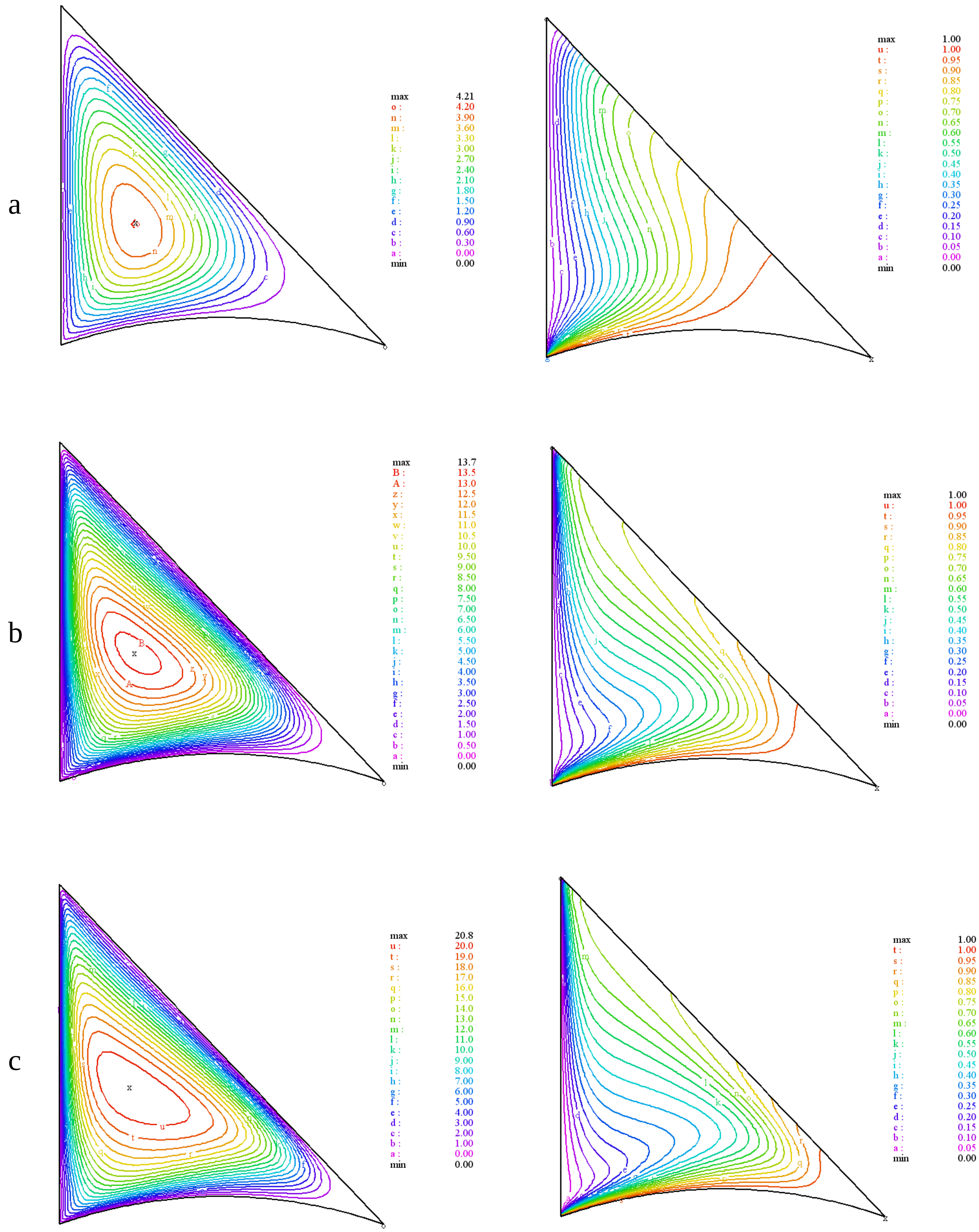

Fig.4 Streamlines (left) isotherms (right) at AR=1 and R=1.5 a) Ra=100 b) Ra=500 c) Ra=1000

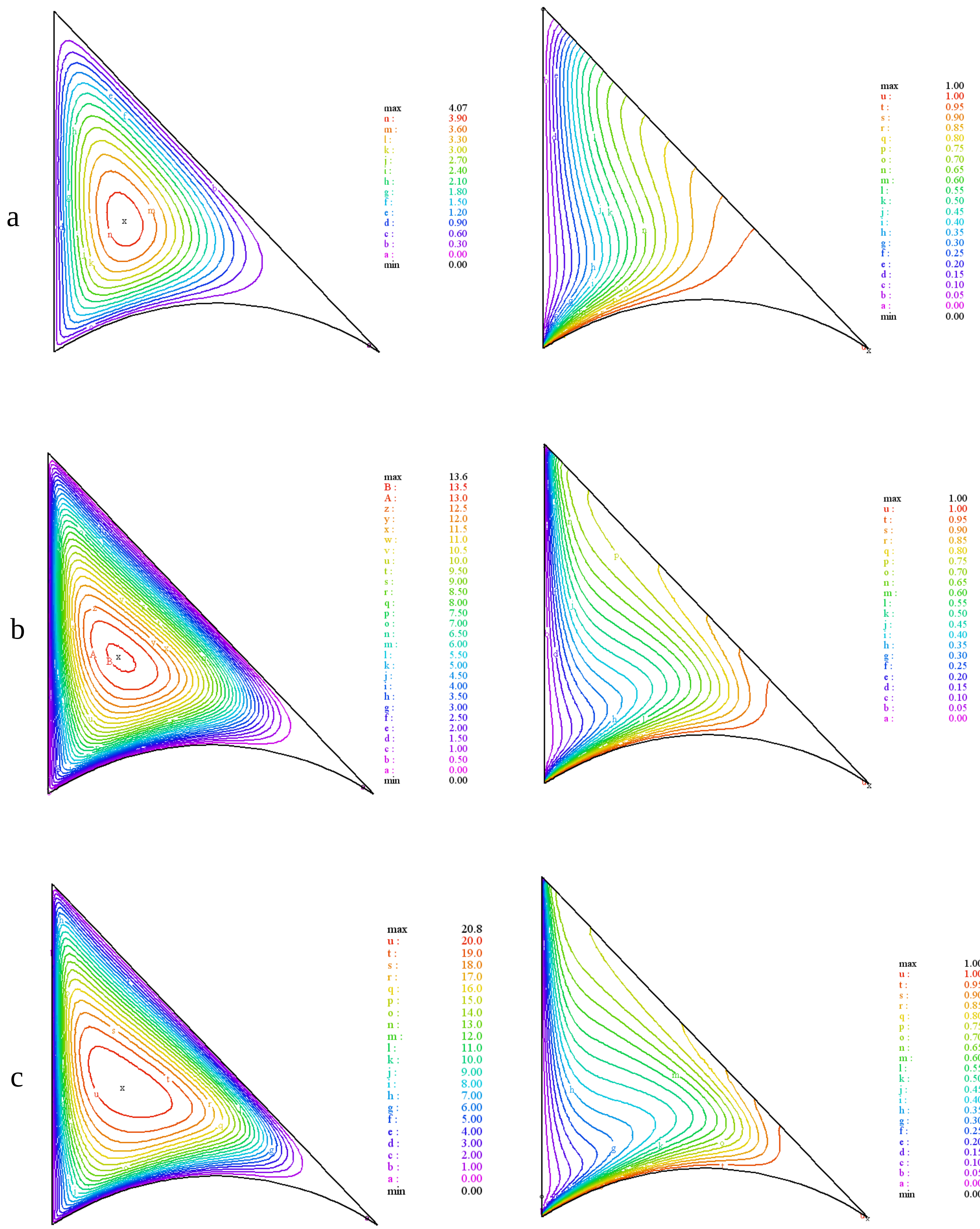

Fig.5 Streamlines (left) isotherms (right) at AR=1 and R=0.8 a) Ra=100 b) Ra=500 c) Ra=1000

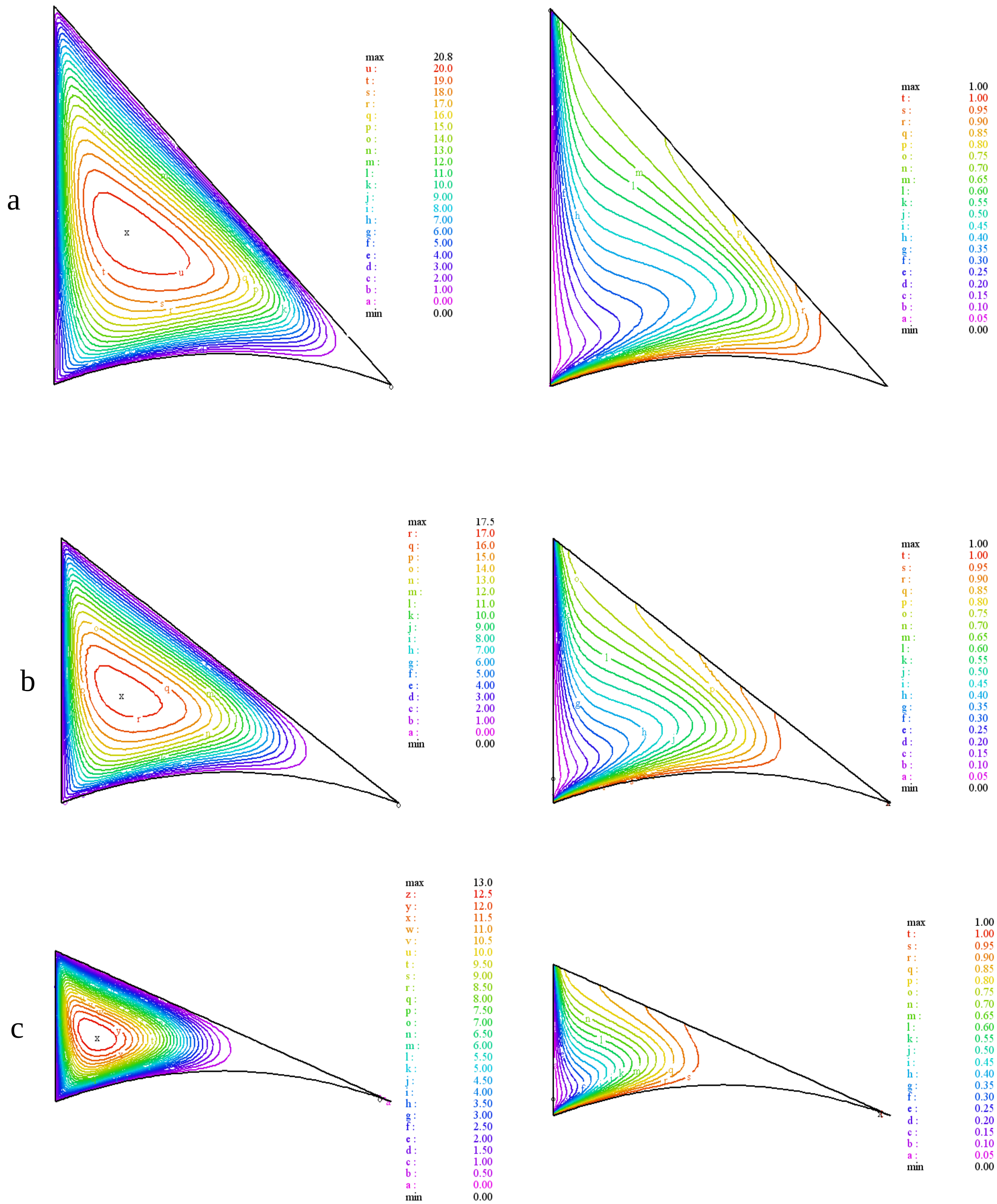

Fig.6 Streamlines (left) isotherms (right) at Ra=1000 and R=1.5 a) AR=1 b) AR=0.7 c) AR=0.4

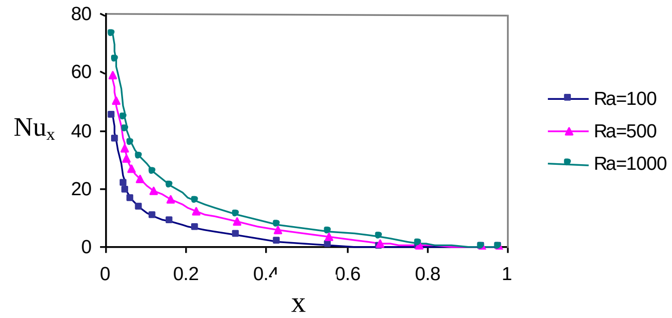

Fig.7 Variation of local Nusselt number along the bottom wall at AR=1 and R=1.5

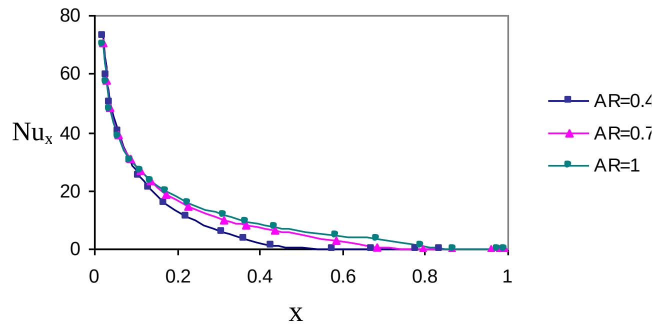

Fig.8 Variation of local Nusselt number along the bottom wall at Ra=1000 and R =1.5

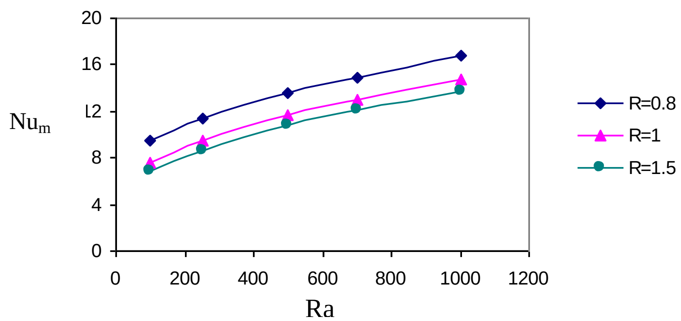

Fig.9 Variation of mean Nusselt number with Rayleigh number at AR=1 and different radius R .

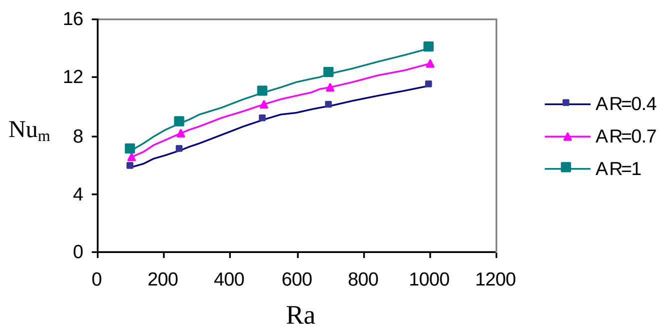

Fig.10 Variation of mean Nusselt number with Rayleigh number at R=1.5 and different aspect ratio .

5. Conclusions

In the present work the curvature of the bottom wall and the aspect ratio represent the problem of natural convection heat transfer in the isothermal triangular enclosure. The results were obtained for different Rayliegh numbers . The important conclusions drawn from this study as follow :

I) Flow fields and isotherms are strongly affected by changing Rayleigh number, aspect ratio and the curvature of the bottom wall..

II) Rayliegh number, aspect ratio and curvature of the bottom wall affect the amount of circulation mass inside the enclosure.

III) Heat transfer is an increasing function of Rayliegh number, aspect ratio and the curvature of bottom wall for all casings .Conduction becomes dominant at small Rayliegh numbers .

IV) The overall heat transfer is significantly enhanced with an increasing Rayleigh number, aspect ratio and increasing the curvature of bottom wall.

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Nomenclature

AR aspect ratio H/L

g gravitational acceleration (m/s^2)

H height of triangular enclosure

K permeability of the porous medium (m^2)

L length of the bottom wall (m)

Nu_L local Nusselt number

Nu_m mean Nusselt number

R non-dimensional radius of the curve of bottom wall (r/L)

Ra Darcy- modified Rayleigh number $Ra=g\beta kL/(T_H-T/\upsilon\alpha)$

T Fluid temperature (K)

u,v dimensional velocity component m/s

U,V non-dimensional velocity component , uL/α , vL/α

X,Y non-dimensional coordinates , $X=x/L$, $Y=y/L$

Greek Symboles

α thermal diffusivity of porous media

β thermal expansion coefficient

θ non-dimensional temperature , $\theta=(T-T_C)/(T_H-T_C)$

υ kinematics viscosity (m^2/s)

Subscripts

C cool
L Local
m mean